A NEW PROCEDURE FOR TWO-PHASE THERMAL ANALYSIS OF MULTI-PASS INDUSTRIAL PLATE-FIN HEAT EXCHANGERS

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ABSTRACT

A simple alternative to the NTU and/or F-factor (MTD) method of analyzing plate-fin heat exchangers is presented in this paper. The new procedure discretizes the flow path in each stream into finite difference-type grid points. Analysis is carried out in the cells defined by adjacent grid points. Defining the geometric and thermal relationships between cells in the two fluid streams is a key to the success of the method. Unlike the procedures that the new one is intended to replace, there are no limitations regarding the flow arrangement, number of passes, banking type, or phase change. Furthermore, judicious choice of the grid points along a passage allows the treatment of strong dependence of property on temperature. Finally, the new procedure is validated by comparing its predictions with the NTU method and experimental measurements of a multi-pass plate-fin heat exchanger. Heat transfer between R134a and a proprietary fluid is calculated with the Kandlikar correlations (Kandlikar, 1990), to demonstrate the capability for multi-pass, two-phase, plate-fin systems.

INTRODUCTION

Plate-fin heat exchanger rating analysis usually involves the determination of the total heat transfer rate, the pressure drop and the outlet temperatures (Anonymous, 1994). In its most basic form, plate-fin heat exchangers involve the transfer of heat between two fluids – termed the hot and cold fluid. The fluids are separated from each other in passages with a large amount of surface area between them. The exchange of heat takes place between fluids moving in parallel or cross-flow directions. Parallel flow arrangements may also be either co-current or counter-current. In the co-current arrangement, both the hot and cold fluids are flowing in the "same" direction. In this case, the hottest section of the hot fluid is exchanging heat with the coldest section of the cold fluid and temperature difference progressively becomes smaller along the flow direction. Counter-current parallel flow is the opposite of this arrangement and the flow of the fluids is in opposite directions. The use of cross counter-current plate-fin heat exchangers is also quite common. Finally, realistic systems often contain multiple passes or they may involve a phase change.

Because of the foregoing, the analysis of realistic heat exchanger systems represents a challenging task. The modern technique of computational fluid dynamics (CFD) (Ladeinde and Nearon, 1997; Shah et al., 2000), although very promising, is not yet at the stage where it can routinely be applied to the analysis of two-phase heat exchanger systems with realistic geometries. Therefore, empirical methods currently provide the means of analysis. The text by Hewitt et al. (1994) provides the basic analytical procedures for thermo-hydraulic performance and design analysis of various types of heat exchangers. By far the two most common empirical approaches are the F-factor and NTU methods (Bowman et al., 1940; Shah and Sekulic, 1997). The F-factor of a given heat exchanger is the ratio of the effective temperature difference in the exchanger to that in a pure counter-current flow arrangement with identical terminal temperatures. Closed-form formulas for F-factors were presented by Bowman et al. (1940) for a few plate-fin arrrangements. For other selected group of heat exchangers, one must calculate the F-factor from NTU and other parameters. The functional relationship is known only for a relatively small selection of configurations and is not by any means explicit because it depends on the unknown outlet temperatures. The NTU method provides curves or functional relationships for the effectiveness, which is the ratio of heat transfer rate obtained from a system to that obtainable from an ideal system operating between the maximum temperature difference.

The F-factor and NTU methods do not offer much help in the analysis of many heat exchanger systems, particularly those involving a phase change and/or configurations for which no correlations or graphs are available. Moreover, these overall methods are restricted to constant properties and overall heat transfer coefficient, which is obviously not the case for phase change duties.

The present paper presents a versatile finite difference-type procedure for single phase and two-phase heat transfer analysis in a multi-pass plate-fin heat exchanger. The procedure is easy to implement and is readily applicable to various plate-fin flow arrangements, including arbitrary cross counter-current flow and cross co-current flow arrangements. The results from the proposed technique and from the NTU method are compared with experimental measurements of a single-phase, multi-pass, plate-fin system. Two-phase calculations with the method are also presented. The experimental measurements were carried out by Lytron, Inc., MA (USA).

NUMERICAL PROCEDURE

Our method is illustrated in this section with a two-stream, plate-fin problem. The flow passages are divided into small sections. The calculation tracks the flow from the inlet to the outlet of a fluid stream as the fluid goes through the passage. The fluid passage being tracked is termed the primary passage while the complementary passage of the other fluid is the secondary passage. In Figure 1, for instance, for every section along the cold fluid passage, there is an exchange of heat with the hot fluid through the thin separating plate. The heat transfer rate is computed in each section of the primary passage. The total heat transfer rate is an aggregate of the values in the various sections that make up the passage. The temperatures and other states of the fluid in the secondary passage are calculated at the end of every iteration.

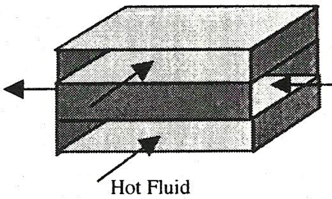


Figure 1. Schematic of the cross flow arrangement in a plate-fin heat exchanger

For plate-fin heat exchangers, heat transfer occurs in its basic form between a hot and cold fluid separated by plates. This is illustrated in Figure 1 for the cross flow arrangement.

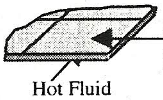


Figure 2. Infinitesimal area of hot and cold fluid exchanging heat

The heat flux between a sub-section of hot and cold fluid shown in Figure 2 can be expressed as

$$q'' = U\Delta T\,,$$

where q'' is the heat flow rate per unit area, ΔT or $(T_h - T_c)$ is the local temperature difference between the fluids, while U is the overall heat transfer coefficient for the small strip, which can be calculated from

$$\frac{1}{UA} = \frac{1}{(A\alpha)_h} + R_w + \frac{1}{(A\alpha)_c}\,, \tag{1}$$

where A is the effective heat transfer surface area which includes the effect of fin area, α is the effective heat transfer coefficient which depends on the Reynolds number, the Prandtl number and fluid fouling factor. Note that R_w is the wall resistance which, for a plate-fin heat exchanger, is given by

$$R_w = \frac{a}{A_w k_p}\,,$$

where a is the plate thickness, A_w is the total wall area for conduction, and k_w is the conductivity of the plate material. Note that $A_w = L_1 L_2 N_p$ where L_1, L_2 and N_p are the length, width, and the total number of separating plates.

From the basic nature of the cell in Figure 2, it is obvious that this technique is independent of whether the flow is cross or parallel. An important aspect of our procedure is the tracking of the flow segments for a variety of flow arrangements. Segments are groups of sections, which are exchanging heat with the fluid on the opposite side of the wall at the same station. In Figure 3, for example, the sections that make up segment 1 of the primary passage, for instance, will exchange heat with the same sections of fluid in the secondary passage at segment 16 of the secondary passage.

Tracking the Flow Segments

For flow through a passage of four passes, the primary passage segments can be easily tracked by a loop. However, the segments of the fluid on the opposite side of the plate are not apparent. This is illustrated in Figure 3.

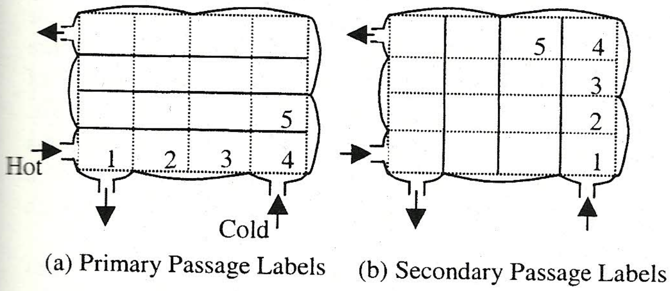


Figure 3. Labeling of the fluid segments for a 4 pass by 4 pass cross counter-current flow heat exchanger

In the example in Figure 3, there are 16 flow segments. Assuming that the hot fluid, with inlet as shown, is the primary fluid, then this fluid will initially exchange heat with the 16th segment of the cold fluid. The temperature difference across a segment is of interest. To calculate the temperature difference across a segment, the temperature of the fluid in the secondary passage must be determined. This will be obtained only after the complementary segment of the secondary fluid corresponding to the primary fluid segment under consideration has been determined. Denoting the segment numbers of the primary fluid by p and that of the secondary fluid by q, the mapping between p and q can be expressed as

$$q = 2N_q \times \mathrm{int}\left(\frac{i}{2}\right) + j \times \mathrm{mod}(i,2) - (j-1) \times \mathrm{mod}(i+1,2)$$

where

N_p = number of passes of the hot fluid
N_q = number of passes of the cold fluid

and (i, j) is a matrix of indices, with i running from right to left in Figure 3 and j from the bottom to the top.

The proposed procedure is described below.

Iterative Procedure

1) The passage for the primary stream is discretized into a total of, say, N_1 sections. The N_1 sections are further grouped into segments, depending on the intersection with the secondary stream. Thus, a segment contains more than one section. Note that segments are labeled *1*, 2, ... in Figure 3.
2) The passage for the secondary stream is also discretized into a total of, say, N_2 sections. The N_2 sections are further grouped into segments, depending on the intersection with the primary stream.
3) An initial guess for the heat flux is made, and a compatible initial temperature distribution is generated.
4) For each section j of the primary passage, the following steps are carried out:
   a) obtaining fluid properties at the current value of the temperature.
   b) updating the flow quality, x, at the downstream end of the section
   c) calculating the temperature for the section using, for illustration only, procedures such as

$$T_{j+1} = T_{sat} \text{ (phase change)}$$

$$T_{j+1} = T_j + \frac{q_j dxW}{\dot{M}C_p} \text{ (single-phase),}$$

   where W is the flow width of the section and dx is the section or strip length
   d) calculating single-phase or two-phase pressure drop, as the case may be
   e) calculating the heat transfer coefficient, α (single-phase or two-phase) for the current section j
   f) computing fin efficiency, $(\eta_f)_j$ and the effective heat transfer area A_j'
   g) for each section k of the secondary fluid which exchanges heat with the current jth section:
      i) determine the segment to which k belongs and hence the temperature of the secondary fluid
      ii) calculate ΔT
      iii) get the fluid property for the secondary fluid at the secondary fluid's temperature
      iv) calculate α, A' and η_f for the secondary fluid
      v) calculate overall U
      vi) calculate $q'' = U\Delta T$.
5) For every section k of the secondary fluid
   a) retrieve the q'' values from Step 4 above
   b) compute the temperature for the section using similar equations to 4(c).
6) Go back to (4) and iterate until convergence.
7) Compute the overall heat transfer rate by summing the section values in the primary or secondary passage. Also compute the overall ΔP by integrating dp/dx across the entire passage.

In item (6) above, convergence is assumed when ε_s, defined as

$$\varepsilon_s = \sum \frac{\delta T_i}{\Delta T},$$

where δT_i is the residual and ΔT is the temperature difference that corresponds to the driving force.

The procedure involves the integration of certain quantities along the flow passage. For example, the pressure loss is given by

$$\int \frac{dp}{dx} dx$$

where dp/dx depends on the passage coordinate x. This integration is carried out using Simpson's 1/3 rule. Other quantities that need to be integrated along x include the U and the total fin efficiency.

RESULTS

The procedure reported in this paper has been incorporated into the general-purpose thermal analysis software called INSTED®. As is apparent from the description above, the procedure is equally applicable to both single-phase and two-phase problems. It is not limited by the number of passes of the hot or cold stream, as the results below illustrate.

Single-phase, Multi-pass Applications

The method described has been used for performance calculations of a plate-fin exchanger for streams of hot oil and air. The results are compared with those obtained from the NTU method and experimental measurements.

The oil has properties ρ = 922.633 kg/m^3 (57.6 lb/ft^3), μ = 4.274x10^{-3} Ns/m^2 (10.342 lb/hrft), k = 0.1267 W/mK (0.0732 BTU/hrft°F), C_p = 2110.86 J/kgK (0.5042 BTU/lb°F). The fin description on either side is outlined in the table below:

Table 1. Fin description for single-phase calculation

	Oil	Air
Fin type	Triangular, with lanced offset	Triangular, louvered
Fin thickness	0.000152m 0.006″	0.000152m 0.006″
Offset pitch	0.003175m 1/8″	0.003175m 1/8″
Fin density	1102 fins/m 28 FPI	452.7 fins/m 11-12 FPI
Fin height	0.0032m 0.126″	0.0106m 0.420″

The number of passes on the oil side is two. Total number of oil passages is 12 while the number of air passages is 13. The core height is 0.04064m (1.6″) with core width of 0.19685m (7.75″). This gives a flow length of 0.3937m (15.50″) for the oil and 0.04064m (1.6″) for air. The separator bar thickness is 0.0094m (0.37″), giving a flow width of 0.01613m (0.635″) for the oil and 0.1872m (7.37″) for air. The plates are made of aluminum with thickness 0.0004064m (0.016″).

Seven combinations of inlet temperatures and flow rates were computed. The results are shown in the Table 3.

Table 2. Process Conditions for the single-phase heat transfer calculations

Case No.	Air Flow	Oil Flow	Air Inlet Temp.	Oil Inlet Temp.
A	0.49 kg/s 3892.1 lb/hr	0.32 kg/s 2539.4 lb/hr	300.61 K 81.41 °F	380.85 K 225.84 °F
B	0.2146 kg/s 1703.3 lb/hr	0.437 kg/s 3472.1 lb/hr	299.054 K 78.61 °F	390.76 K 243.69 °F
C	0.44 kg/s 3487.0 lb/hr	0.4363 kg/s 3462.8 lb/hr	300.61 K 81.41 °F	384.83 K 233.01 °F
D	0.489 kg/s 3881.3 lb/hr	0.4375 kg/s 3472.1 lb/hr	300.8 K 81.74 °F	383.22 K 230.12 °F
E	0.443 kg/s 3515.0 lb/hr	0.32 kg/s 2539.4 lb/hr	302.17 K 84.22 °F	383.89 K 231.31 °F
F	0.212 kg/s 1683.0 lb/hr	0.32 kg/s 2539.4 lb/hr	298.81 K 78.17 °F	385.08 K 233.45 °F
G	0.44 kg/s 3498.1 lb/hr	0.32 kg/s 2539.4 lb/hr	300.4 K 81.02 °F	381.81 K 227.57 °F

Table 3. Results of single-phase heat transfer calculations compared with experiments

Case No.	Q	ε	Q	ε	Q	ε
	(NTU)		(New Method)		Experiment	
A	8725 W 29773.5 Btu/hr	0.22	9486 W 32368.4 Btu/hr	0.24	10607W 36197.3 Btu/hr	0.27
B	8509.6W 29037.3 Btu/hr	0.43	9057.9W 30908.3 Btu/hr	0.457	8132W 27726.6 Btu/hr	0.41
C	10959W 37396.1 Btu/hr	0.29	11939W 40742. 6 Btu/hr	0.321	11048W 37701.0 Btu/hr	0.30
D	11207W 38241.7 Btu/hr	0.276	12772W 43581.7 Btu/hr	0.315	11431W 39006.5 Btu/hr	0.28

E	8605W 29365.9 Btu/hr	0.236	9375.4W 31991.9 Btu/hr	0.257	10289W 35109.8 Btu/hr	0.28
F	6684.9W 22810.9 Btu/hr	0.363	7271.8W 24708.8 Btu/hr	0.373	7350W 25080.7 Btu/hr	0.40
G	9214.7W 31443.4 Btu/hr	0.255	10015W 34175.3 Btu/hr	0.277	10214W 34855.7 Btu/hr	0.28

The temperature variation along the length of both passages for Case A, non-dimensionalized by each stream's flow length is shown in Figure 4.

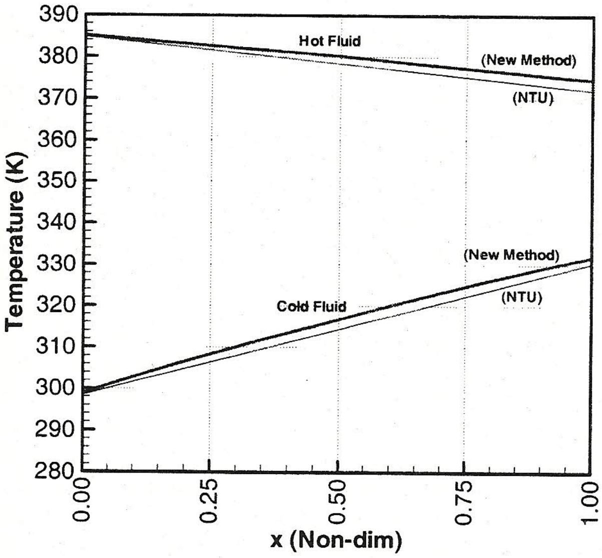


Figure 4. Temperature profile for single-phase heat transfer calculation

Deviation from a linear temperature profile is evident in Figure 4. The NTU method gives a straight-line temperature profile in its normal usage, although it is possible to employ the method in a temperature-dependent framework. However, the beauty of the new method lies in the natural way in which property variation along the heat exchanger is handled and the generality with respect to phase change, number of passes, flow arrangement and banking. The NTU and F-value methods are proven methods and Table 3 shows that they provide fairly accurate results. The results also show that the new method is quite competitive. Note that the motivation for the new method is in the ability to analyze new designs for which no F-value or empirical effectiveness relations are available.

Application to Two-Phase Heat Transfer Between R134a and a Proprietary Fluid

The method described in this paper has been used to compute the heat transferred between a hot fluid and R134a. The hot fluid has the properties: ρ = 1818 kg/m^3 (113.5 lb/ft^3), μ = 0.00455 Ns/m^2 (11 lb/hrft), k = 0.0066 W/mK (0.0381 BTU/hrft°F), C_p = 900.11 J/kgK (0.215 BTU/lb°F). The description of the fins on either side is outlined in the table below:

Table 4. Fin description for two-phase calculation

	Hot Fluid	R 134a
Fin type	Triangular, with lanced offset	Triangular, with lanced offset
Fin thickness	0.000152m 0.006″	0.000152m 0.006″
Offset pitch	0.003175m 1/8″	0.003175m 1/8″
Fin density	709 fins/m 18 FPI	709 fins/m 18 FPI
Fin Height	0.002184m 0.086″	0.002184m 0.086″

The number of passes of the hot fluid is four and there are 44 passages. The number of refrigerant passes is two; the number of passages is 20. The core length is 0.2286m (9″) with core width of 0.05715m (2.25″). This gives a flow length of 0.05715m (2.25″) for the hot fluid and 0.2286m (9″) for the refrigerant. The separator bar thickness for both streams is 0.003175m (0.125″), giving a flow width of 0.0532m (2.094″) for the hot fluid passage and 0.0238m (0.9375″) for the refrigerant passage. The plates are made of aluminum with thickness 0.0004064m (0.016″). The flow rate of the refrigerant is 0.0019656 kg/s (156 lb/hr) and that of the hot fluid is 1.053 kg/s (8358 lb/hr). The inlet quality of the refrigerant is 10%. Several methods were used to calculate α for sections undergoing two-phase heat transfer but the results shown are for the boiling procedures in Kandlikar (1990). The two-phase pressure drop was calculated using the Martinelli procedure.

Calculations using the method outlined in this paper give a total heat flow rate of 2802 W (9561 Btu/hr) with an exit quality of 78% for the refrigerant. The temperature variation along the length of both passages, non-dimensionalized by each stream's flow length, is shown in Figure 5.

Comparison of the two-phase calculations with limited experimental measurements shows good agreement.

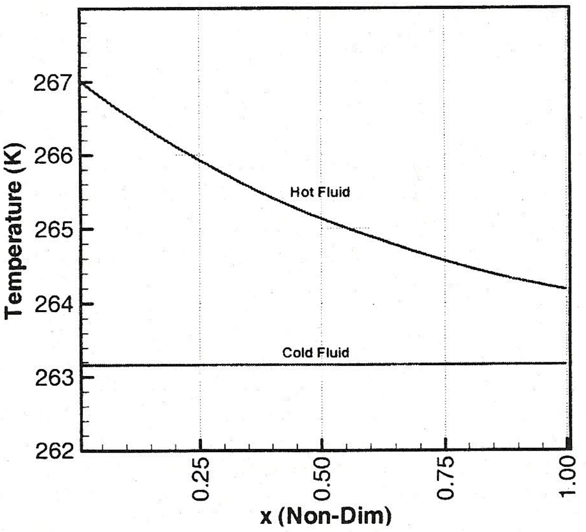
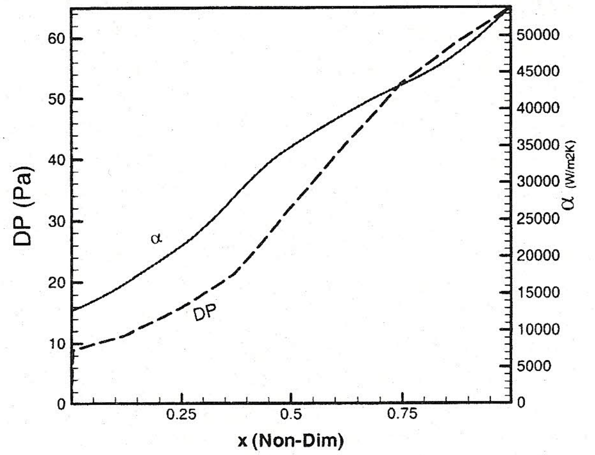


Figure 5. Temperature profile for two-phase heat transfer calculation

Figure 5 shows a nonlinear temperature variation for the hot fluid while the temperature of the refrigerant is fairly constant, indicating that the passage is not long enough to cause a sensible heat transfer in the refrigerant.

Figure 6. Profile of the pressure drop and the heat transfer



coefficient, α, along the cold stream passages

Figure 6 represents a plot of the pressure drop and heat transfer coefficient along the cold fluid passages. The pressure drop increases along the passages. This is expected because of the dependence of Δp on quality, which increases from the inlet value of 10% to 78% at the outlet.

Figure 6 also shows a significant increase in the two-phase heat transfer coefficient, α, as the passage is traversed. This can be explained in terms of the increasing quality, which appears explicitly in the convection number, *Co*:

$$Co = \left(\frac{1-x}{x}\right)^{0.8}\left(\frac{\rho_g}{\rho_l}\right)^{0.5}.$$

Note that in the Kandlikar procedure, *Co* is raised to the power of a negative number.

CONCLUSIONS

In this paper a method is presented for thermo-hydraulic analysis of plate-fin heat exchangers. This method is presented as an alternative to the F-factor and NTU methods. The new method is not limited by phase change, strong temperature dependence of properties or the number of passes. The method is also readily applicable to various flow arrangements and single and multiple banking.

To validate the procedure, computed results are compared with experimental measurements for single-phase and boiling in plate-fin heat exchangers.

Further development of the procedure reported in this paper is being undertaken to address the potential problems that might compromise the robustness of the method. For example, when both streams involve two-phase flow (such as evaporation on the cold side and humid air on the hot side), the numerical stability of the method becomes an issue. In this case, the heat transfer coefficient depends on the heat flux in both streams. There are ways to resolve this stability issue, however.

The paper assumes uniform distribution of the inlet fluid into the passages. A mal-distribution situation can also be analyzed although the specifics of the distribution has to be provided to the program.

The present version of the program assumes the saturation temperature is a function of some average pressure. The case of dependence with the local segment pressure complicates the procedure somewhat, although calculations are still possible.

NOMENCLATURE

a	plate thickness, m
A'	effective heat transfer area, m^2
A	area of plate, m^2
C_p	specific heat, J/kgK
h_{lg}	latent heat, J/kg
k_p	plate thermal conductivity, W/mK
$\dot{M}$	mass flow rate, kg/s

$N_{passages}$	number of passages
q''	heat flux, W/m^2
R	wall resistance, K/W
T	temperature, K
U	overall heat transfer coefficient, W/m^2K
ρ	density, kg/m^3
α	heat transfer coefficient, W/m^2K

Subscript

i	stream
h	hot fluid
c	cold fluid
g	gas
l	liquid
1	primary
2	secondary

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